

STRUCTURES

Deflection of Beams



MATRIX

CP1879

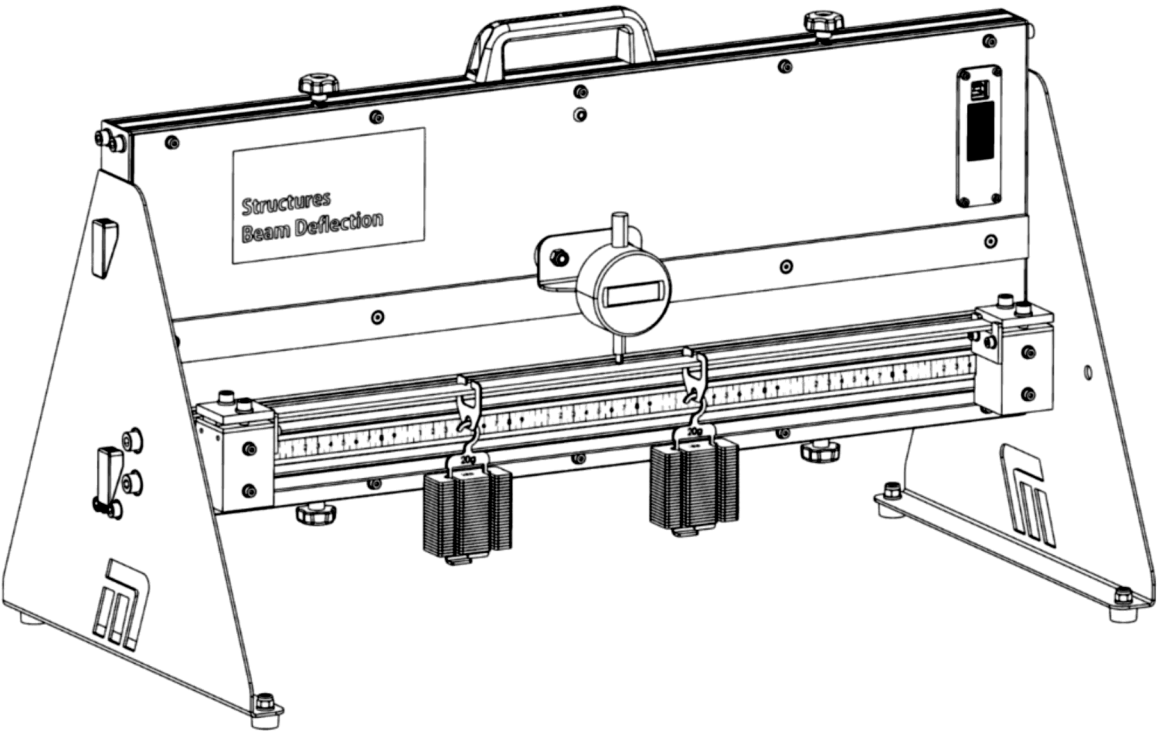
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Deflection of a beam

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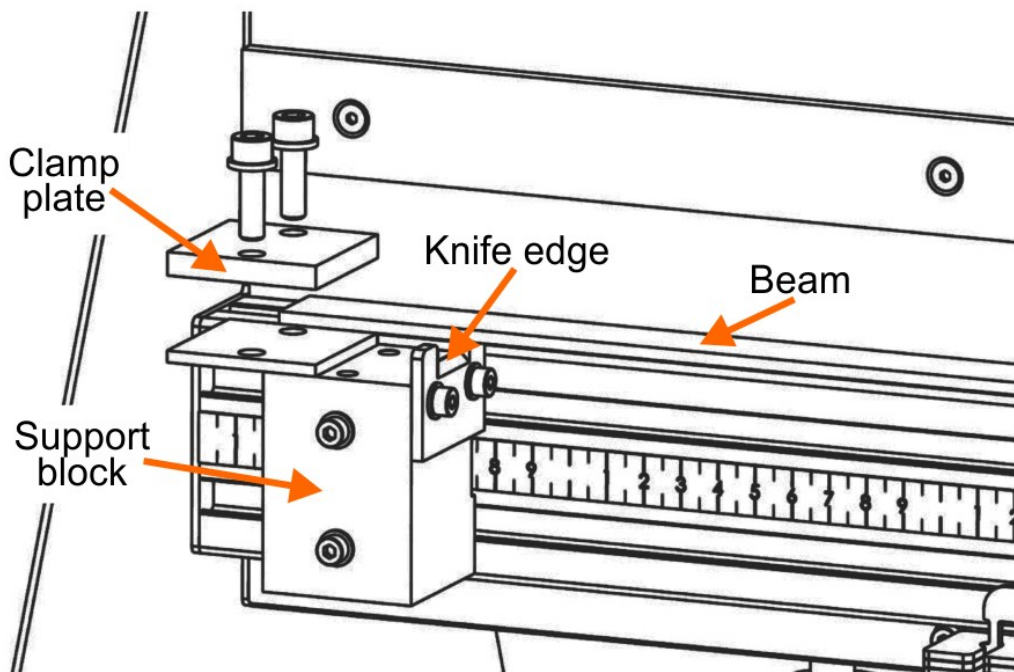
Introduction



The apparatus:

The apparatus is designed to work off 5v power supply. This means that a USB cable plugged into either a computer or a plug will be sufficient. The data acquisition software only works through the computer, therefore the recommended setup is to have the USB plugged into the computer which is running the software. However, if you'd like to run the experiment without the software, a USB plug will need to be sourced for the correct local plug style.

Close-up view of the beam support mechanism



Loosen the support block bolts, to slide the support left and right to change distance.

Remove clamp plates to take away the fixed support option.

Slide the knife edge plate up and down to add the simply support with the 2 bolts.

Simple supports:

- Set up a knife-edge support by:
 - removing the screws holding the clamp plate to the support block;
 - lifting off the clamp plate;
 - loosening the bolts on the side of the support block and sliding up the knife-edge holder, until the knife edge protrudes above the edge of the block.
- Loosen the bolts on the front of the support block slightly and slide the block to the side to the desired position.

Fixed supports:

- Remove the knife-edge plate by retracting it fully.
- Place the beam in position over the support block.
- Replace the spacer and clamp plate and tighten them evenly on both sides of the beam.

Dial gauge:

- Attach the dial gauge to the bracket and fix it over the centre of the beam so that its lower tip rests lightly on it.
- The magnets on the dial gauge holder can be moved sideways to adjust the position of the dial gauge.

Precautions:

- Zero' the readout on the dial gauge using the buttons on the instrument to eliminate the masses of the beam and empty mass hanger in later calculations.
- Tap the bench gently to reduce the effect of friction on the deflection.

Worksheet 1

Effect of load

This investigation looks at how much a beam bends when subjected to different loads.

For the engineer, this can be a vital aspect of design - in bridges, buildings, machinery etc.

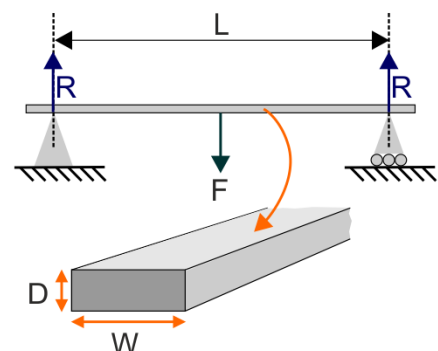
The photograph shows part of a girder used in a bridge.

The curved shape is designed to make it more rigid when loaded and to simplify the way in which it is supported at its ends.



Over to you:

- Set up simple supports at both ends of the apparatus, separated by a distance of 400mm.
- Use a calliper gauge to measure the dimensions of aluminium beam 1 and then place it centrally across the supports.
- Record your measurements in the Student Handout.
- Clamp the beam in position with the clamp plates.
- Suspend one empty mass hanger under the beam, mid-way between the support blocks, i.e. 200mm from each.
- 'Zero' the dial gauge.
- Add a 100g mass to the mass hanger.
- Observing the precautions listed earlier, use the dial gauge to measure the deflection of the beam.
- Convert this into metres, for later calculations and record it in the table in the Student Handout or use the USB data link to log it in an Excel spreadsheet.
- Repeat this process, increasing the load on the beam in steps of 100g up to 500g.
- Plot a graph of deflection vs load and measure its gradient, following the instructions in the Student Handout.
- The free-body diagram for this arrangement is shown opposite:



Worksheet 1

Effect of load



So what:

- This arrangement simulates a simply supported beam.
- The theoretical deflection δ is given by the equation:

$$\delta = \frac{F \cdot L^3}{48 E \cdot I}$$

where **F** is the weight of the load on the beam,
L is the length of the beam,
E is the Young's modulus of the material in the beam,
I is the area moment of inertia of the beam.

- For a rectangular cross-section bar:

$$I = \frac{W \times D^3}{12}$$

Since **L**, **W**, **D** and **E** are constant for this beam,

$$\delta = \text{constant} \times F \quad \text{where this constant} = \frac{L^3}{48 \times E \times I}$$

Hence, a graph of deflection δ vs load **F** should be linear.

- Use your results, with the deflection measured in millimetres, to plot this graph.
- Calculate the gradient **m** of this straight line in $\text{m} \cdot \text{N}^{-1}$ (for use in later worksheets.)

Worksheet 2

Effect of material

When designing structures, knowing the properties of the materials involved is especially important.

These may be their thermal properties, like melting point and conductivity, mechanical properties, like hardness, density and ductility or chemical properties, like flammability and reactivity.



This investigation looks at the effect of some of the mechanical properties on the deflection produced by a given load, for samples of different metals with similar dimensions and then uses the results to estimate one of these properties, Young's modulus.

Over to you:

- Repeat the procedure outlined in worksheet 1 for:
 1. the brass beam;
 2. the steel beam.
- Record all results in the tables in the Student Handout or in an Excel spreadsheet, using the USB data link to log them.
- Use the results to plot straight-line graphs of deflection δ vs load F for the brass beam and the steel beam.
- Measure their gradients and record the values in the Student Handout.

Worksheet 2

Effect of material



So what:

Finding Young's modulus

Option 1:

For a beam loaded at its centre, resting on two pinned supports, (one effectively being a roller support as well), the beam equation is:

$$\text{deflection } \delta = \text{constant} \times F \quad \text{where this constant} = \frac{L^3}{48 \times E \times I}$$

In other words, a graph of deflection δ vs load F is linear with a gradient $m = \frac{L^3}{48 \times E \times I}$

Hence,
$$\text{Young's modulus } E = \frac{L^3}{48 \times m \times I}$$

- Use the gradients of the three graphs - aluminium (from worksheet 1), brass and steel, to calculate values for Young's modulus for the three metals.
- Record your answers in the Student Handout.

Option 2:

For a beam loaded at its centre:
$$\text{deflection } d = \frac{F \times L^3}{48 \times E \times I}$$
 where d is the deflection in m.

- In the Student Handout, re-arrange this formula to show that a graph of F vs $(48 \times d \times I) / L^3$ is linear with a gradient equal to Young's modulus.

For each metal:

- use the measurements made in worksheets 1 and 2, for aluminium, brass and steel, to complete the table of results in the Student Handout.
- plot a graph of F vs $(48 \times d \times I) / L^3$ for each metal.
- By measuring the gradient of each graph, obtain values for Young's modulus for the three metals.
- Record your answers in the Student Handout.

Worksheet 2

Effect of material



And finally:

The brass and steel beams have similar dimensions, **L**, **D** and **W**, to those for the aluminium beam used in worksheet 1.

Hence, they all have similar values for **I**, the area moment of inertia.

The formula on page 7 shows that deflection δ for a given force **F** is proportional to $1 / E$, where **E** is Young's modulus for the material.

In other words, **for a given load**,

$$\delta (\text{brass}) / \delta (\text{aluminium}) = E (\text{aluminium}) / E (\text{brass})$$

and

$$\delta (\text{steel}) / \delta (\text{aluminium}) = E (\text{aluminium}) / E (\text{steel})$$

and

$$\delta (\text{steel}) / \delta (\text{brass}) = E (\text{brass}) / E (\text{steel})$$

- Using your measurements, calculate the following ratios, **for a load of 5N:**
 - deflection of brass / deflection of aluminium,
 - deflection of steel / deflection of aluminium,
 - deflection of steel / deflection of brass.

and write the answers in the Student Handout.

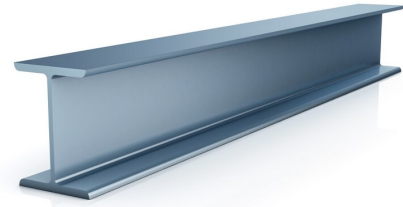
- Compare these ratios with the ratios of the metals' Young's modulus given in the Student Handout.

Worksheet 3

Effect of dimensions



When designing a structure, one consideration is the material to use in its construction. Young's Modulus indicates the tensile and compressive stiffness of the material, allowing the relative strengths of materials to be compared.

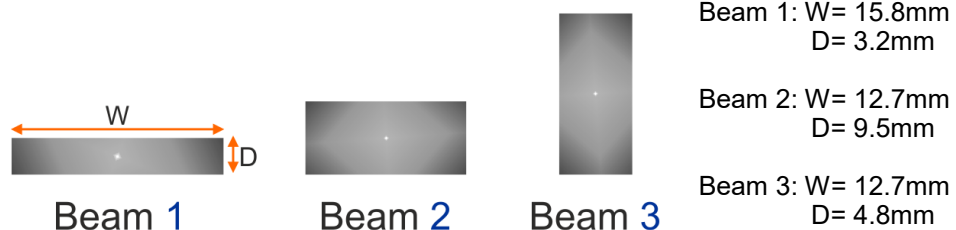


In this experiment, three beams of the *same* material, aluminium, and *same* length are subjected to the *same* loading while resting on the *same* supports.

The only difference is in the dimensions of the beams. This results in different values for their area moments of inertia.

Differences in the deflections produced must be the result of changing this factor.

The diagram shows the three aluminium beams in the kit.



Over to you:

- Use a calliper gauge to measure the dimensions, W and D , of the beams and record them either in the tables in the Student Handout or in an Excel spreadsheet.
- Calculate and record the area moment of inertia for each beam, using the formula:

$$I = W \cdot D^3 / 12$$

- Position each one in turn centrally on the supports.
- Suspend a mass of 200g from the centre of the beam.
- Measure and record the deflection produced, observing the usual precautions.
- Plot a graph of deflection vs $1/I$ and answer the question posed in the Student Handout.

So what:

- The depth of the beam has a greater effect on the deflection value than its width because of the D^3 term, which gives beam 2 and beam 3 very different I values, even though they are the width.

Worksheet 4

Effect of supports



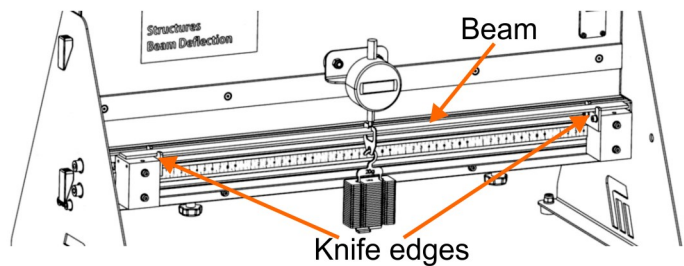
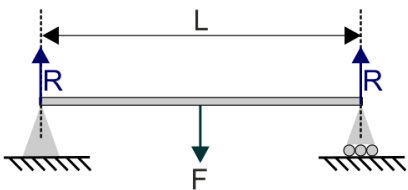
The way in which the beam is supported affects how much it bends (and the beam deflection equation!)

The investigations, so far, have used only simple supports, which provide only vertical reaction forces.

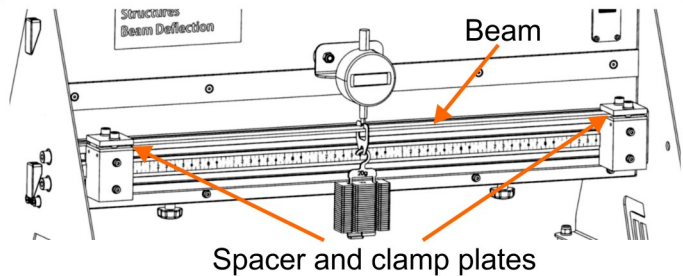
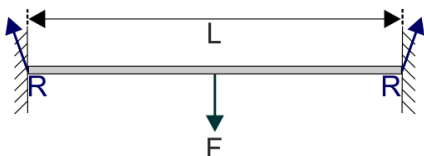
The focus of this investigation is to look at two other types of support as well, fixed supports and propped cantilever supports.



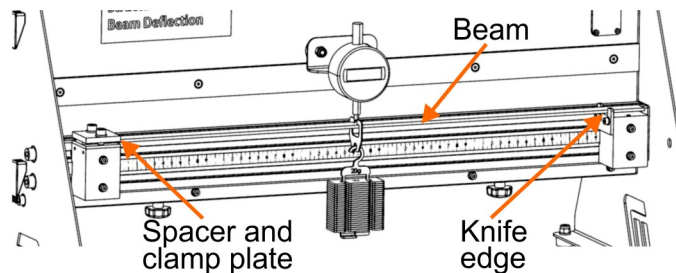
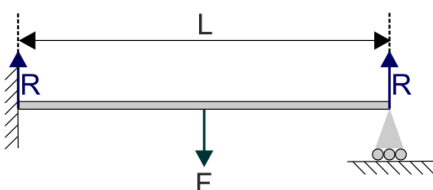
Simple supports:



Fixed supports:



Propped cantilever supports:



The propped cantilever beam is a horizontal beam with a fixed support at one end and a roller support at the other.

Worksheet 4

Effect of supports



Over to you:

Challenge:

- Using the aluminium beam 1, investigate the effect of the three types of support on the deflection produced when the beam is under load and use the results to obtain the value of **K**, the constant in the general formula for the deflection of a beam loaded at its centre, for each type of support.

- Consider:
 - what parameters to keep constant throughout the investigation and what to vary;
 - what parameters you need to measure and over what range;
 - how many measurements you need to reach a firm conclusion;
 - how to display your results to justify a conclusion.

- Using either tables drawn in the Student Handout or an Excel spreadsheet, record your results.

- In the Student Handout:
 - list the results of your considerations;
 - present the results of your investigations;
 - give the conclusion(s) you reached;
 - show how your results justify these conclusions.

Worksheet 5

Effect of length

Worksheet 3 investigated the effect of the width and depth of a beam on how much it bends under load.

Now it's time to consider the effect of length. Obviously these factors are linked. A short, thick beam will bend less under a given strain than a long thin one.

The photograph shows a situation where it is important to know how long the crane arm can be extended without it bending too much when it lifts a heavy load.

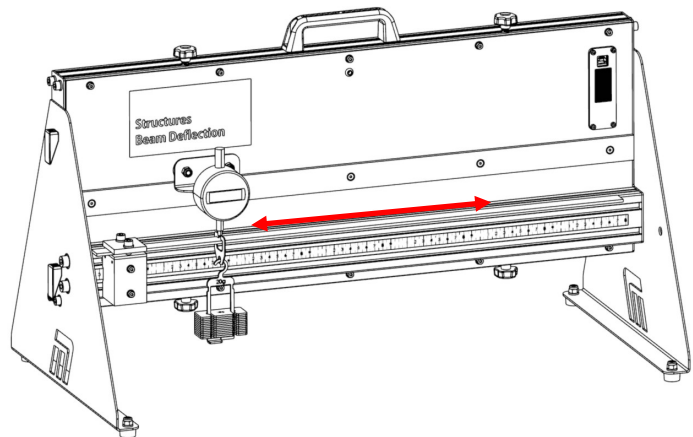


Over to you:

Challenge:

Set up the steel beam as a cantilever by clamping one end and leaving the other unsupported.

Move a constant load along the beam and investigate the link between the distance from the fixed end to the load and the deflection produced.



- Consider:
 - what parameters are kept constant and what vary;
 - what parameters you need to measure and over what range;
 - how many measurements you need to reach a firm conclusion;
 - how to display your results to justify a conclusion.
- Using either tables drawn in the Student Handout or an Excel spreadsheet, record your results.
- In the Student Handout:
 - list the results of your considerations;
 - present the results of your investigations;
 - give the conclusion(s) you reached;
 - show how your results justify these conclusions.



Student Handout

Worksheet 1 - Effect of load

Aluminium beam 1:

Length **L** of beam =mm =m

Width **W** of beam =mm =m

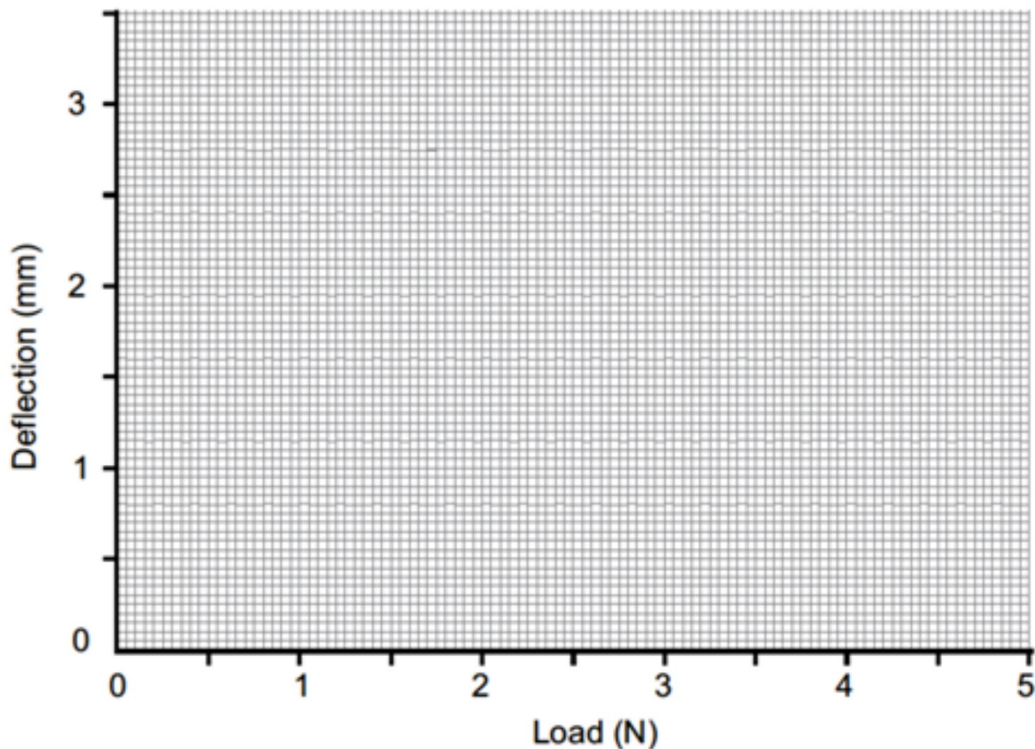
Depth **D** of beam =mm =m

Suspended mass in g	Load F in N	Deflection of beam δ in mm
0	0	0
100	1	
200	2	
300	3	
400	4	
500	5	

Graph of deflection vs load:

Show your measurements as small crosses.

They should suggest a straight-line relationship.



Gradient **m** of graph =mm.N⁻¹ =m.N⁻¹

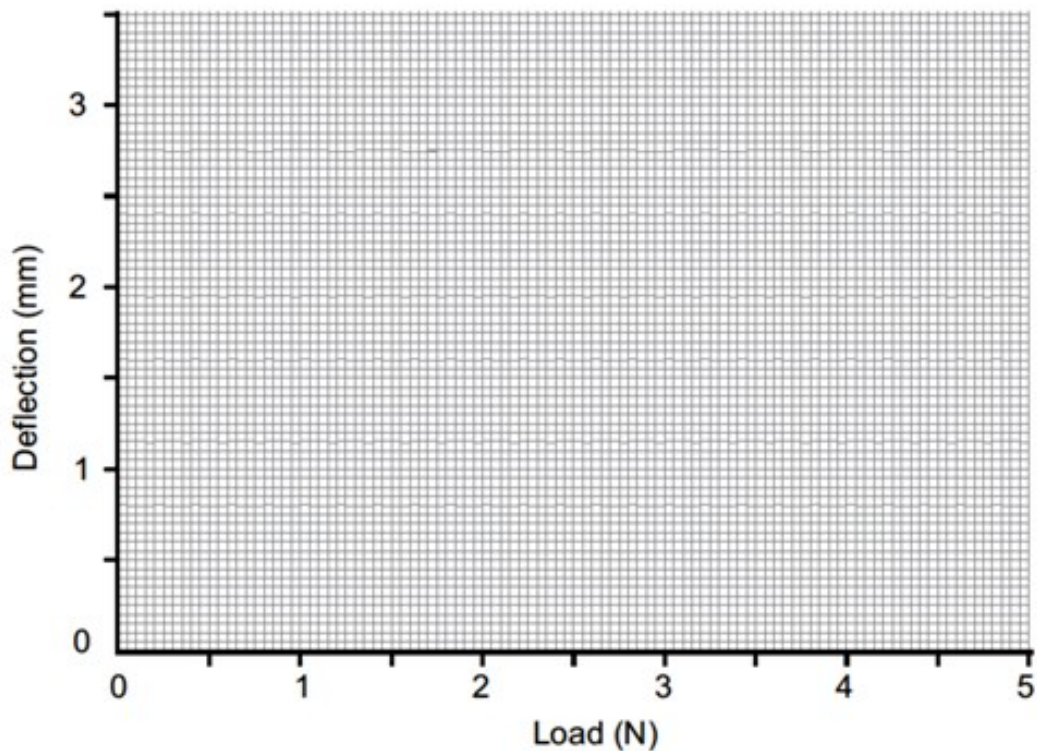
Worksheet 2 - Effect of material

Brass beam: Length **L** of beam =mm =m
 Width **W** of beam =mm =m
 Depth **D** of beam =mm =m

Suspended mass in g	Load F in N	Deflection of beam δ in mm
0	0	0
100	1	
200	2	
300	3	
400	4	
500	5	

Graph of deflection vs load:

Show your measurements as small crosses. They should suggest a straight-line relationship.



Gradient **m** of graph =mm.N⁻¹ =m.N⁻¹

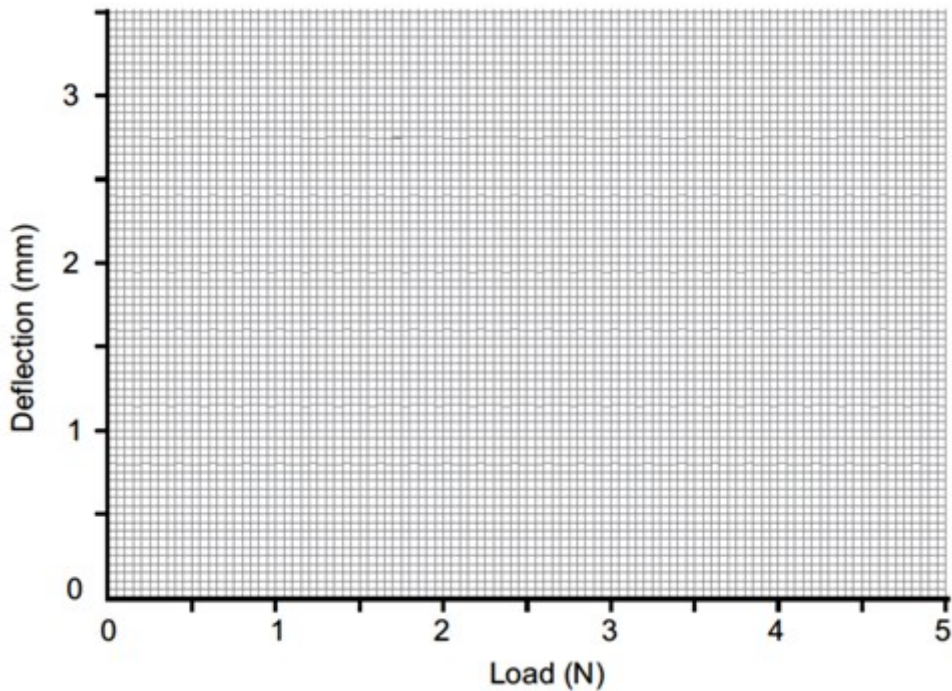
Worksheet 2 - Effect of material

Steel beam: Length **L** of beam =mm =m
 Width **W** of beam =mm =m
 Depth **D** of beam =mm =m

Suspended mass in g	Load F in N	Deflection of beam δ in mm
0	0	0
100	1	
200	2	
300	3	
400	4	
500	5	

Graph of deflection vs load:

Show your measurements as small crosses. They should suggest a straight-line relationship.



Gradient **m** of graphmm.N⁻¹ =m.N⁻¹

Worksheet 2 - Effect of material

Option 1:

Brass -

Area moment of inertia of cross-section $I = \frac{W \times D^3}{12} = \dots\dots\dots$

Young's modulus for brass $E = \frac{L^3}{48 \times m \times I} = \dots\dots\dots$

Steel -

Area moment of inertia of cross-section $I = \frac{W \times D^3}{12} = \dots\dots\dots$

Young's modulus for steel $E = \frac{L^3}{48 \times m \times I} = \dots\dots\dots$

Aluminium -

Area moment of inertia of cross-section $I = \frac{W \times D^3}{12} = \dots\dots\dots$

Young's modulus for aluminium $E = \frac{L^3}{48 \times m \times I} = \dots\dots\dots$

Option 2:

For a beam loaded at its centre: deflection $d = \frac{F \times L^3}{48 \times E \times I}$ where d is the deflection in m.

Re-arrange this to show that a graph of F vs $(48 \times d \times I) / L^3$ is linear with a gradient equal to Young's modulus.

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Worksheet 2 - Effect of material

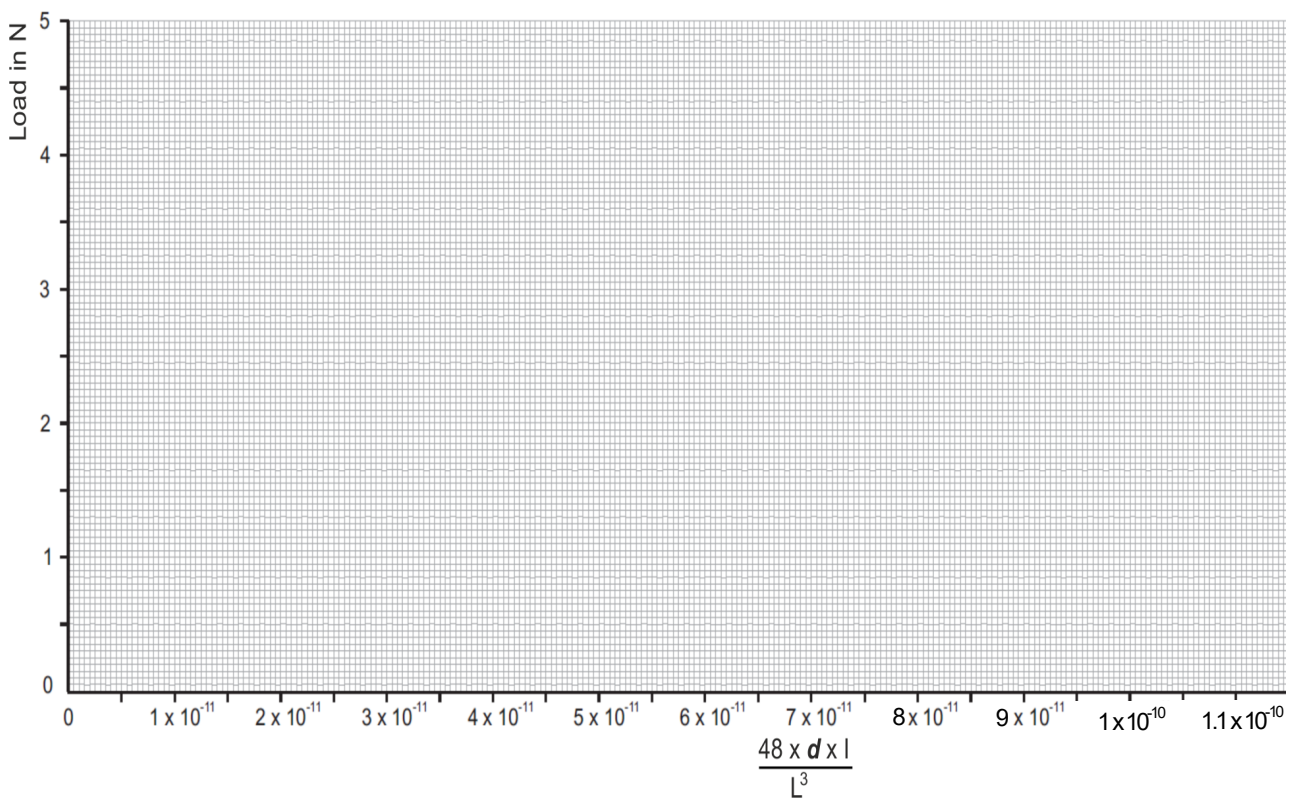
Option 2:

Metal

Suspended mass in g	Load F in N	Deflection of beam δ in mm	Deflection of beam d in m	$48 \times d \times l / L^3$
0	0	0	0	
100	1			
200	2			
300	3			
400	4			
500	5			

Graph of $(48 \times d \times l) / L^3$ vs load:

Show your measurements as small crosses.



They should suggest a straight-line relationship.

Worksheet 2 - Effect of material

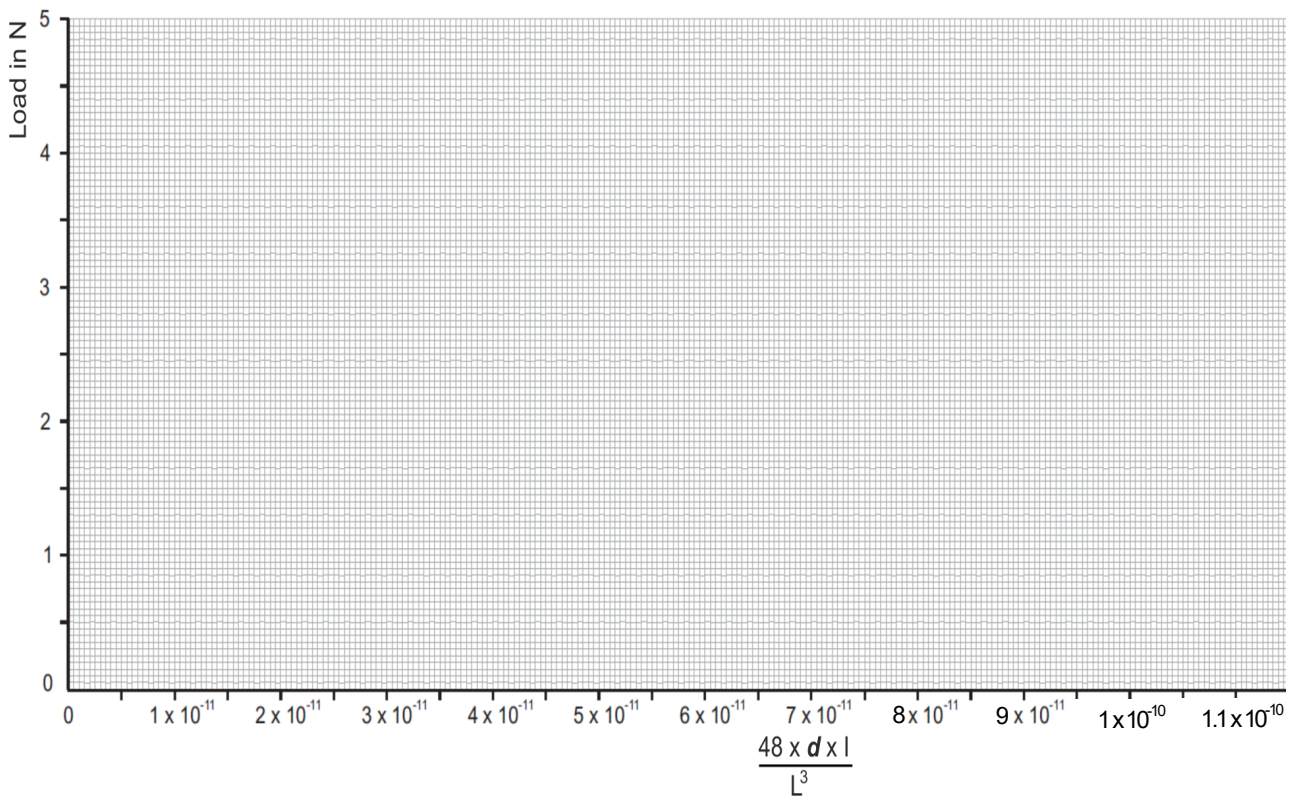
Option 2:

Metal

Suspended mass in g	Load F in N	Deflection of beam δ in mm	Deflection of beam d in m	$48 \times d \times l / L^3$
0	0	0	0	
100	1			
200	2			
300	3			
400	4			
500	5			

Graph of $(48 \times d \times l) / L^3$ vs load:

Show your measurements as small crosses.



They should suggest a straight-line relationship.

Worksheet 2 - Effect of material

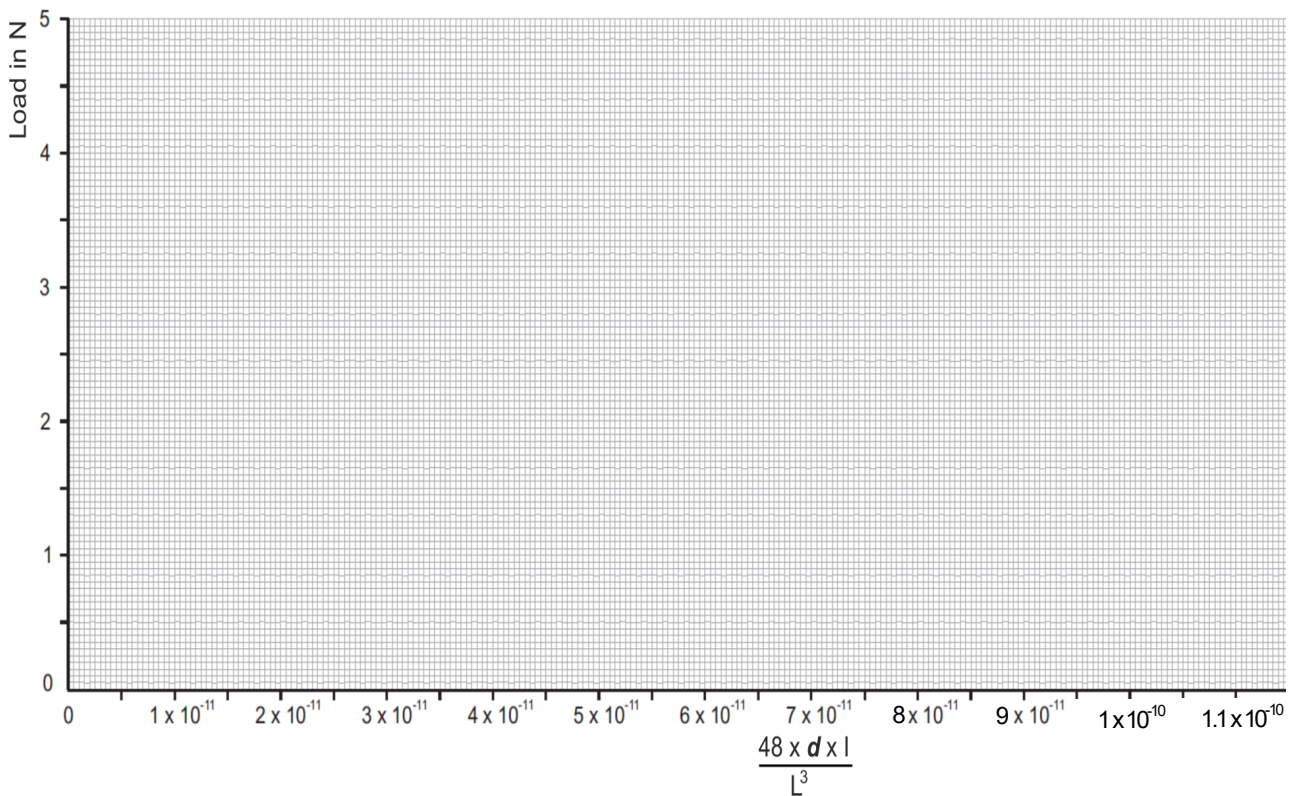
Option 2:

Metal

Suspended mass in g	Load F in N	Deflection of beam δ in mm	Deflection of beam d in m	$48 \times d \times l / L^3$
0	0	0	0	
100	1			
200	2			
300	3			
400	4			
500	5			

Graph of $(48 \times d \times l) / L^3$ vs load:

Show your measurements as small crosses.



They should suggest a straight-line relationship.

Worksheet 2 - Effect of material

And finally:

Data:

For a load of 5N:

deflection of aluminium = (taken from worksheet 1 results.)

deflection of brass = (taken from worksheet 2 results.)

deflection of steel = (taken from worksheet 2 results.)

Calculations:

Using these:

$$\frac{\text{deflection of brass}}{\text{deflection of aluminium}} = \dots\dots\dots$$

$$\frac{\text{deflection of steel}}{\text{deflection of aluminium}} = \dots\dots\dots$$

$$\frac{\text{deflection of steel}}{\text{deflection of brass}} = \dots\dots\dots$$

Using these values:

$$\frac{E(\text{aluminium})}{E(\text{brass})} = \dots\dots\dots \quad \frac{E(\text{aluminium})}{E(\text{steel})} = \dots\dots\dots \quad \frac{E(\text{brass})}{E(\text{steel})} = \dots\dots\dots$$

The table opposite shows typical values of Young's modulus, **E**, for the three metals.

Compare these with the ratios of deflections calculated above and comment on the comparison:

Metal	Young's modulus E in GPa
Aluminium	69
Brass	105
Steel	200

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Which of the measurements you made is likely to cause the largest error in the calculation of Young's modulus? Give a reason for your choice.

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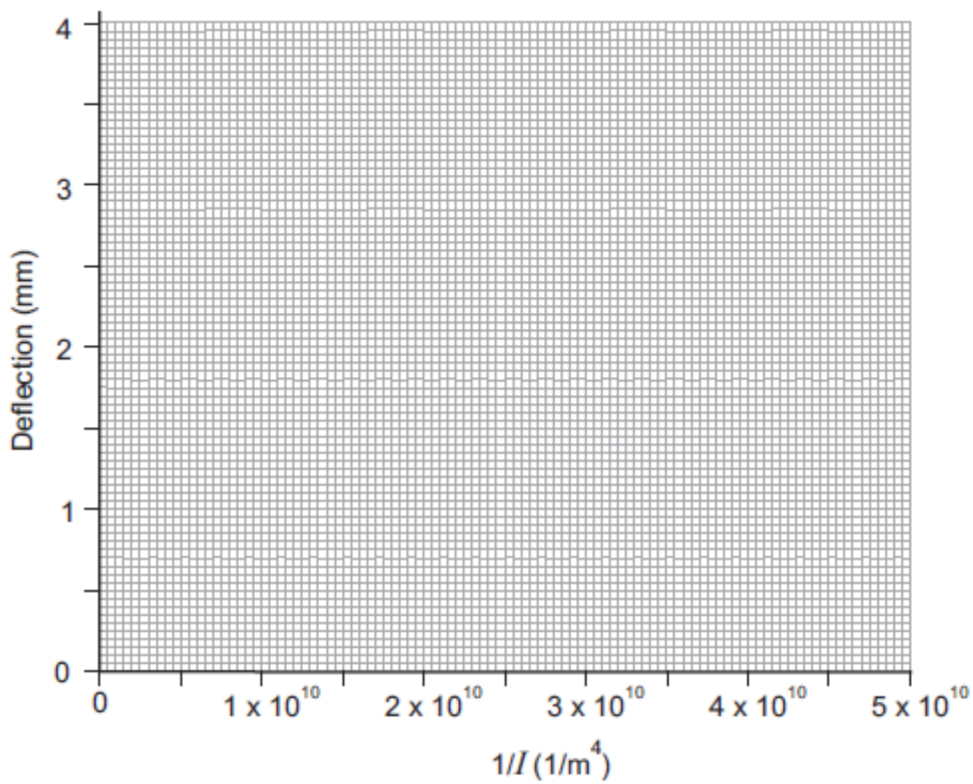
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Worksheet 3 - Effect of dimensions

Sample	Width W in m	Depth D in m	Area moment of inertia in m^4	Deflection δ in mm
1				
2				
3				

For a load of 2N:

Graph of deflection δ vs $1/I$:



Theory predicts that the relationship between deflection and area moment of inertia should be linear and pass through the origin.

Suggest reasons why this might not be the case for measured values:

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Student Handout



Worksheet 4 - Effect of supports

Use the following space to draw tables for your results if you wish.

Student Handout



Worksheet 5 - Effect of length

Use the following space to draw tables for your results if you wish.

Worksheet 5 - Effect of length

- List the results of your considerations;
- Give the conclusion(s) you reached;
- Show how your results justify these conclusions.

A series of horizontal dotted lines providing space for writing answers to the worksheet questions.

Notes for the Instructor

About this course

Introduction

This module allows students to investigate the effects of forces generated in simple beam structures through a structured sequence of practical investigations.

Using the kit, students complete a series of worksheets that focus on a number of topics found in BTEC Higher National and equivalent courses.

Aim

The course teaches students about the relationships between and effects of the forces arising in simple beam structures and their supports.

Prior Knowledge

It is expected that students have followed an introductory science course, enabling them to take, record and analyse scientific observations and appreciate the errors inherent in them. Some mathematical capability is required.

Using this course:

It is expected that the Worksheets and Student Handout are printed / photocopied, preferably in colour, for the students' use.

Each worksheet includes:

- an introduction to the topic under investigation;
- step-by-step instructions for the investigation that follows.

The Student Handout is a record of measurements taken in each worksheet and questions relating to them. Students do not need a permanent copy of the worksheets but do require their own copy of the Student Handout

This format encourages self-study, with students working at a rate that suits their ability. It is for the instructor to monitor that their understanding is keeping pace with progress through the worksheets. One way to do this is to 'sign off' each worksheet, as the student completes it, and in the process have a brief chat to assess the student's grasp of the ideas involved in the exercises it contains.

We realise that you as a subject area practitioner take the lead in determining how and what students learn. The worksheets are not meant to supplant this or any other supporting underpinning knowledge you choose to deliver. For subject experts, the 'Notes for the Instructor' are provided simply to reveal the thinking behind the approach taken.

For staff whose core subject knowledge is not in the field covered by the course, these notes can both illuminate and offer guidance.

Time:

It will take students between three and five hours to complete the worksheets. It is expected that a similar length of time will be needed to support the learning that takes place as a result.

Learning Objectives

On successful completion of this course, the student will be able to:

- use vernier callipers to measure the dimensions of a metal bar;
- use a dial gauge to measure the deflection of a beam;
- use the formula 'deflection $\delta = \frac{FL^3}{48EI}$ ' to explain why the graph of δ vs F should be linear;
- explain the meaning of the term Young's modulus;
- outline three ways in which Young's modulus can be obtained from the graph of δ vs F ;
- re-arrange the formula 'deflection $\delta = \frac{FL^3}{48EI}$ ' so that the subject of the formula is load F ;
- compare the measurements made to deduce which errors have the greatest significance on the experimental value of Young's modulus;
- explain in qualitative terms the way in which the profile of a beam affects its rigidity;
- design an experiment to compare the rigidity of beams of the same metal having different profiles;
- explain why the depth of the beam is more significant than its width in determining its rigidity;
- distinguish between the following types of support:
 - simple;
 - fixed;
 - cantilever;
 - propped cantilever;
- design an experiment to investigate the effect of the three types of support on the deflection produced when the beam is under load;
- use the results to obtain the value of K , the constant in the general formula for the deflection of a beam loaded at its centre, for each type of support;
- give three examples of common structures that use a cantilever beam;
- design an experiment to investigate the link between the distance to the load and the resulting deflection for a cantilever structure and use the results to establish and justify the relationship between them.

Worksheet	Notes
<p>1 Effect of load</p> <p>Timing 20 - 30 mins</p>	<p>Concepts involved: simple support weight mass gravitational field strength linear relationship</p> <p>When setting up the equipment, careful positioning and adjustment of the dial gauge is very important.</p> <p>Students may need an introduction to the use of vernier callipers to enable them to measure beam dimensions and to the use of the USB link to download data from the dial gauge.</p> <p>Depending on their mathematical ability and experience, the students may need help in transposing formulae and also with the standard equation for a straight-line graph.</p>
<p>2 Effect of material</p> <p>Timing 40 - 60 mins</p>	<p>Concepts involved: Young's modulus area moment of inertia</p> <p>The practical work mirrors that in the previous investigation but uses brass and steel beams instead of aluminium.</p> <p>The main issue is the way in which the results are processed. The worksheet lists three options. The instructor could delegate different options to different groups.</p> <p>The third option needs an understanding of proportionality and may require support from the instructor.</p>
<p>3 Effect of profile</p> <p>Timing 30 - 50 mins</p>	<p>There are no new concepts in this investigation.</p> <p>The techniques are the same as those used in previous practical work. The area moment of inertia depends on the profile of the beam and so can vary for the same beam depending on how it is used. The instructor may wish to reinforce the point made about the significance of the depth of the beam.</p> <p>This is a good point for a discussion about reading errors.</p>
<p>4 Effect of supports</p> <p>Timing 40 - 60 mins</p>	<p>Concepts involved: fixed supports propped cantilever supports</p> <p>This is an appropriate point at which to compare the degrees of freedom and reaction forces offered by the different types of support. There may need to be an initial discussion on the factors to consider in designing a 'fair' experiment.</p> <p>The numerical value in the denominator of the deflection formula depends on the type of support used, but can be obtained from a graph, as described here.</p>
<p>5 Effect of length</p> <p>Timing 30 - 50 mins</p>	<p>Concepts involved: cantilevers</p> <p>The students design an investigation into the link between the distance from the fixed end to the load and the deflection produced in a cantilever beam. They could be asked to justify their approach to other groups in a class discussion.</p>